Interaction dynamics of X-waves in waveguide arrays

Matthew O. Williams, Colin W. McGrath, J. Nathan Kutz
Department of Applied Mathematics, University of Washington, Seattle, WA 98195
mowill@amath.washington.edu

Abstract: The interaction dynamics of X-waves in an AlGaAs waveguide array is theoretically considered. The nonlinear discrete diffraction dynamics of a waveguide array mediates the generation of spatio-temporal X-waves from pulsed initial conditions. The interactions between co-propagating and counter-propagating X-waves are studied. For the co-propagating case, the initial phase relation between the X-waves determine the attractive or repulsive behavior of the X-wave interaction. For the counter-propagating case, the collisions between X-waves generate a nonlinear phase-shift. These dynamics show that X-waves interact in a manner similar to solitons.

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References and links
1. Introduction

The study of X-waves was first introduced in the context of linear phenomenon. The so-called X-waves were localized solutions of linear wave equations in the diffraction- and dispersion-free limit [1, 2]. Despite its genesis in linear theory, X-waves have emerged as a general model capable of describing nonlinear phenomena in settings whose underlying physics is described by a linear (hyperbolic) Schrödinger operator [3]. Thus the application of X-waves to optical systems is not surprising given the central role of the linear Schrödinger operator in characterizing laser beams subject to diffraction and normal group-velocity dispersion (GVD) in bulk. Indeed, envelope X-waves were first observed in second-harmonic generation [4] and later extended to explain dynamic filamentation [5] as well as parametric generation in water [6]. X-waves also play a relevant role in periodic media such as photonic crystals or Bose condensed gases (where a negative effective mass due to a periodic potential mimics normal GVD) [7, 8] and have been observed in discrete systems [9] such as waveguide arrays (WGA) [10, 11]. The ubiquitous nature of X-waves also suggests that they have applications outside of conservative models. Indeed, such X-wave patterns have also been predicted in dissipative systems [11, 12].

Given the number of nonlinear systems in which X-waves play a role, whether X-waves are linear or nonlinear objects is still a topic of current debate and interest. A recurring issue in this debate is that X-waves are spatio-temporal structures, but they are not stationary or periodic in terms of the propagation distance. Thus, as the X-wave evolves forward in time (or propagation distance) the nonlinear strength of the X-wave decreases. After sufficiently long times, this decrease reduces the impact of the nonlinearity and the X-wave becomes a linear object. However, before the amplitude of the X-wave is sufficiently reduced, nonlinearity plays an important role in its dynamics.

In this manuscript, we study the case where the nonlinearity plays a dominant role and show that there is a certain symmetry between the understanding of solitons and X-waves by comparing the interactions of X-waves to the interactions of solitons. Solitons are fully nonlinear solutions that interact linearly upon collision except for the inclusion of nonlinear phase-shifts and time-lags [13]. Further, whether solitons attract or repel is dependent upon the relative phase-difference between the solitons [13]. To date, the interactions between pairs of X-waves have not been considered. In this manuscript, a variety of X-wave interactions are characterized in the ideal optical setting of a waveguide array (WGA) [14]. Much like solitons, the theoretical study shows that the nonlinear X-wave interactions produce what appear to be linear collision dynamics aside from nonlinear phase changes that are intensity dependent. In the context of co-propagating X-waves, the initial phase difference determines an effective attraction or repulsion dynamics. Thus, the spatial-temporal X-waves behave in much the same manner as solitons.

The paper is outlined as follows: In Sec. 2, the interaction dynamics of both co-propagating X-waves and counter-propagating X-waves are considered. Each of these systems has a different governing set of equations that are formulated in the corresponding subsections. Section 2 represents the critical findings of the paper. Section 3 provides concluding remarks following the numerical findings.

2. Nonlinear X-wave Interactions

The interactions between co-propagating X-waves and counter-propagating X-waves are considered in the context of an experimentally realizable AlGaAs WGAs [14]. The parameters used in simulations are physically relevant for a 3 mm long AlGaAs WGA with input pulses generated from a mode-locked laser producing FWHM pulsewidths of 200 fs [10]. The linear coupling coefficient is taken to be $c = 0.82 \text{ mm}^{-1}$ and the nonlinear self-phase modulation parameter is taken to be $\gamma = 3.6 \text{ m}^{-1}\text{W}^{-1}$. Typical peak powers in the WGA are on the order of kilowatts, and the total number of waveguides is 41 [10, 14]. Because of the parameters and
initial conditions used, trivial modifications to recent experiments [9, 10, 14] should suffice to confirm the numerical simulations presented here.

2.1. Co-propagating X-waves

The first type of X-wave interaction is between co-propagating X-waves. In this interaction, pulses are injected into different but nearby waveguides in the WGA, and the leading-order equations governing the nearest-neighbor coupling (discrete diffraction [14]) of electromagnetic energy in the WGA are given by

\[ i \frac{dA_n}{dZ} + \chi (A_{n-1} + A_{n+1}) + \gamma |A_n|^2 A_n = 0, \tag{1} \]

where \( A_n \) represents the electric field envelope in the \( n \)-th of the \( 2N + 1 \) waveguides \( (n = -N, \cdots, -1, 0, 1, \cdots, N) \) where \( N = 20 \) for 41 waveguides. This set of governing equations has been shown to accurately reproduce experimental findings for pulses with kilowatt peak powers and pulsewidths of hundreds of femtoseconds [10].

To begin, two pulses are launched into two adjacent waveguides with

\[ A_0(0, T) = \eta_0 \text{sech}(T) \text{ and } A_1(0, T) = \eta_1 \text{sech}(T + \Delta T) \times \exp(-i\Delta \theta). \tag{2} \]

and \( A_n(0, T) = 0 \) for \( n \neq 0, 1 \) where \( \eta_0, \eta_1 = 2.0 \) and \( \Delta T = 1 \). At this value, fully nonlinear X-waves are formed during the propagation in the WGA. The dynamics of the X-waves depend on the relative phase difference between the injected pulses. Figure 1 demonstrates a time-history of propagation of (1) with an initial phase difference of \( \Delta \theta = 0 \). From the pulse-initial condition, the characteristic X-wave structure forms. Although the majority of the energy in the X-wave remains confined in the initial waveguide, the low amplitude sections interact and perturb the X-waves. Ultimately, a pair of X-waves results with a slightly larger separation than the original pair of pulses, \textit{i.e.} they repel. In contrast, consider the initial conditions in (2) with identical amplitudes but with \( \Delta \theta = \pi \). A time-history of (1) with these initial conditions is shown in Fig. 2. The WGA still generates an X-wave from each of the initial pulses as shown in the second panel in Fig. 2. However, the separation in \( T \) between the resulting X-waves is negligible, \textit{i.e.} they attract. Therefore, for X-waves with identical amplitudes, the resulting dynamics depends on the initial phase-difference between the pulses.

However, if pulses of different amplitude are launched, the pulse with the largest peak power dominates the interaction. Figure 3 shows the interaction dynamics of two X-waves with initial
conditions given by Eq. (2) where \( \eta_0 = 2.0 \) and \( \eta_1 = 1.0 \) and \( \Delta T = 1 \) and \( \Delta \theta = \pi \). Unlike the equally sized pulses, the larger pulse dominates the dynamics and incorporates the smaller pulse into a single X-wave structure regardless of the phase difference. Therefore, it is the amplitude difference and not the phase-difference that determines the resulting dynamics.

To summarize: the interactions between high amplitude X-waves are nonlinear in nature and exhibit soliton-like dynamics [13]. The dynamics of two co-propagating X-waves is dependent upon both the relative phase-difference and the absolute difference in amplitude of the two injected pulses. Therefore, the X-waves in this situation are not linear objects and share many of the qualities of solitons when they are placed in close proximity.

2.2. Counter-propagating X-waves

Another interaction of interest is the interaction between identical but counter-propagating X-waves. This situation could be generated experimentally by butt-coupling input fibers to opposite ends of a waveguide. The governing equations for counter-propagating waves must include both the forward- and backward-propagating fields. The governing equations, (1), must be modified to account for the second field yielding

\[
\begin{align*}
\frac{i}{\gamma} \frac{dA_n}{dZ} + i\sigma \frac{dA_n}{dT} & + \gamma \left( |A_n|^2 + 2|B_n|^2 \right) A_n + c(A_{n-1} + A_{n+1}) = 0 \quad (3a) \\
\frac{i}{\gamma} \frac{dB_n}{dZ} - i\sigma \frac{dB_n}{dT} + \gamma \left( 2|A_n|^2 + |B_n|^2 \right) B_n + c(B_{n-1} + B_{n+1}) = 0 \quad (3b)
\end{align*}
\]

Fig. 2. Pseudo-color plot of out of-phase pulses interacting. In this case, the X-waves attract and form a pair of X-waves with a negligible delay in time. The white and green dotted lines denote the center of X-wave in the 0th and 1st waveguides at \( Z = 0 \), and the dashed lines represent the center of the X-wave at the present value of \( Z \). (Media 2)

Fig. 3. Pseudo-color plot of two unequally sized X-waves with \( \Delta \theta = 0 \). The X-wave in waveguide 0 has amplitude 2.0 while the pulse in waveguide 1 has amplitude 1. The larger pulse dominates the dynamics and the attracts the smaller pulse. (Media 3)
where $A_n$ is the forward-propagating field of the $n$th waveguide and $B_n$ is the backward-propagating field of the $n$th waveguide. In these equations, the nearest-neighbor coupling and self-phase modulation terms are retained from the co-propagating case, but an additional cross-phase modulation term appears along with a group-velocity term determining the forward ($+$) and backward ($-$) directions of propagation.

The collision of counter-propagating X-waves can be accomplished by launching initial pulses on both sides of the waveguide simultaneously. Thus the initial conditions take the form:

$$A_0(Z,0) = \eta_+ \sech(Z + \Delta Z) \quad \text{and} \quad B_0(Z,0) = \eta_- \sech(Z - \Delta Z).$$

where $2\Delta Z$ measures the initial spatial separation between the right moving (forward-propagating) and left moving (backward propagating) X-waves. It must be stressed that unlike the co-propagating interaction the governing equations of this system occur in a stationary lab frame and the initial condition is defined for all $Z$ at $T = 0$ and not in the usual optical coordinate system. In this case, the waves collide at $Z = 0$. Figure 4 demonstrates the basic collision dynamics. The two X-waves pass through each other without visible deformation. Thus the collision appears to be linear. However, there is an induced nonlinear phase shift due to the collision. The resulting phase shift depends upon the initial launch intensity of the counter-propagating pulses.

Figure 5 demonstrates the phase shift incurred by the X-waves as a function of initial launch intensity. To calculate the phase-shift and time-lag, two simulations were performed. The first simulation includes both the forward- and backward-propagating X-waves. The second simulation only included the forward-propagating X-wave. Both simulations used the same physical and computational parameters as well as the same initial condition for the forward-propagating X-wave. Indeed, the only difference is the presence of the backward-propagating X-wave. After the interaction occurs, the cross-correlation of the forward-propagating X-wave in the 0th waveguide was taken. The maximum of the cross-correlation determines both the time-lag of the X-wave and the relative phase difference between them. It is clear from Figure 5 that X-waves act as linear waves for sufficiently low initial intensities. The nonlinear phase shift approaches zero as the initial pulse heights decrease. As the injected pulses are made more intense, the nonlinearities begin to exhibit themselves and the phase-shift increases in a mono-
tonic fashion. As exhibited in Figure 4 however, the interaction still appears to be linear to the eye. Indeed, regardless of the pulse height there is no observable spatial lag generated by this interaction.

In the case of counter-propagating X-waves, the nonlinearity is difficult to detect because it is not evident in the intensity of X-wave. As demonstrated, counter-propagating X-waves interact in a fashion that does not produce visible changes in the X-wave structure or a measurable delay in the X-wave itself. Nonetheless, the presence of a nonlinear phase-shift that depends on the initial pulse amplitude demonstrates that these X-waves indeed interact in a nonlinear fashion and can be likened to the nonlinear phase shifts experienced by solitons upon collision [13].

3. Conclusions

To conclude, we emphasize that nonlinear X-waves have been typically considered as normal GVD light-bullets generated by initial Gaussian wave-packets. However, the current manuscript shows new aspects to the X-wave dynamics. Specifically, we have provided the first numerical studies of X-waves dynamics under collision and interaction in a waveguide-array model that can be easily implemented experimentally [10]. The results show that the X-waves, whose genesis is in linear theory, behave much like solitons, whose genesis is in a fully nonlinear theory. For co-propagating waves, the X-wave interactions can be attractive or repulsive when the two pulses are out-of-phase or in-phase respectively, similar to solitons [13]. Attractive waves coalesce in time and space while repulsive X-waves separate further than their initial starting value. Counter-propagating X-waves collide leaving little evidence of nonlinear interaction. Indeed, only an intensity dependent nonlinear phase shift is generated. This nonlinear phase shift is much like soliton collisions. However, unlike soliton collisions, no timing shift is measured during the collision process.

The analysis of X-wave interaction, which is based on the quantitative model of waveguide arrays, predicts that the X-wave interactions can be verified with currently available, state-of-the-art technology. The results lead to a better understanding of multi-dimensional self-organized localized structures and to a more fundamental understanding of X-waves as fully nonlinear structures that behave more like solitons than linear structures.

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