Abstract: Spatial mode-locking in three dimensions can be achieved in a slab waveguide array architecture. This study focuses on using the resulting robust and self-starting light bullet formation for photonics applications. Specifically, light bullets can be manipulated through a simple electronically addressable spatial gain dynamics. By applying gain ramps in time and/or space via electronics technology, complete control and manipulation of the light bullets can be achieved, thus allowing for the construction of the master logic gates of NAND and NOR. Its robustness, self-starting behavior and easy addressability suggest that the slab waveguide array mode-locking merits serious consideration as a next generation photonics device.

References and links
12. See the Fundamentals, Functionalities, and Applications of Cavity Solitons (FunFACS) webpage for a complete overview of current and potential methods and realizations of generating localized optical structures: www.funfacs.org.


29. E. J. Doedel, A. R. Champneys, T. F. Fairgrieve, Y. A. Kuznetsov, B. Sandstede, and X. Wang, “AUTO 97: Continuation And Bifurcation Software For Ordinary Differential Equations (with HomCont),”.


1. Introduction

Due to their technological feasibility and inherent nonlinear properties, semiconductor waveguide arrays (WGAs) are an ideal technology for all-optical signal processing applications. Indeed, the experimental demonstrations in 1998 by Eisenberg et al. [1] showed that a sufficiently intense electric field could generate a nonlinear self-focusing effect capable of overcoming the linear, discrete spatial diffraction in the WGAs, thus verifying the early theoretical predictions of Christodoulides and Joseph [2]. Upon experimental demonstration, the WGAs were proposed as ideal components for all-optical routing and switching purposes in, for example, fiber optic networks [1, 3–5]. Such technological promise was not limited to optical routing and switching applications. Specifically, the potential for spatial- or temporal-optical soliton formation was realized early on where the Kerr nonlinearity can be balanced by either spatial diffraction [1–5] (spatial optical solitons) or chromatic dispersion in the context of mode-locked lasers [6–9] (temporal optical solitons). In the latter application, the discrete diffraction plays the crucial role of providing the requisite intensity discrimination (also known as saturable absorption) necessary for mode-locking. In this manuscript, higher-dimensional spatial confinement is achieved with a planar WGA architecture, thus producing a mechanism for light-bullet formation [10–12]. We further propose simple technological enhancements to this configura-
Fig. 1. Schematic of the planar waveguide array. The guiding regions, shown in red, are coupled via evanescent coupling. A gold layer deposited on the 0th waveguide allows for current injection into that waveguide [28]. The gold layer is partitioned into regions. This allows for current to be injected in a non-uniform fashion to different regions of the plane.

tion which allow for complete spatial-temporal control of the light bullets and the construction of photonic devices based upon the WGA structure.

Due to the ridge structure of most WGs, optical solitons are traditionally confined in the direction of (temporal solitons [6–9]) or orthogonal to (spatial solitons [1–5]), the propagation. However, a WGA constructed from three planar waveguides is theoretically capable of producing light bullets that are confined in all three spatial dimensions [13]. The VCSEL-like structure, shown in Fig. 1, forms a light-bullet due to a balance of diffraction and the Kerr nonlinearity in the plane of the waveguide and is assumed to be a Bragg or gap soliton in the propagation direction [14–16]. While this is certainly not the only method for the formation of light bullets [10–12, 17–19] nor is it the only technology capable of controlling the bullets [20–23] (e.g. using a holding beam [24–27]), this form of the WGA offers a flexible, highly-robust, and electronically controllable method for the routing and control of light bullets.

Starting from a cold cavity, the device in Fig. 1 is capable of producing stable and robust light bullets given the right parameter values [13]. In [13], it was assumed the level of current injection was uniform across the entire device. This creates a translational invariance in the system and, as a result, any bullet created will not move in the plane. Intuitively, this invariance means all locations on the array are the same and there is therefore no reason for bullet motion. The initial formation of the light bullet is determined from the initial noise fluctuations in the cavity or gain medium. In this manuscript, the assumption of spatially uniform gain is relaxed, and gain is now allowed to become spatially nonuniform and time varying. A nonuniform gain is indeed an experimentally obtainable result. The amount of gain given to any one region on the waveguide can be controlled by dividing the waveguide into discrete regions, as shown by the black grid-lines in Fig. 1, and pumping each of the regions separately. With modern electronics the level of current pumping, and therefore gain, can be changed temporally as well. Thus, the spatial and temporal non-uniformity discussed in this manuscript is not only a theoretical construct, but is also physically realizable with modern experimental techniques. This allows for complete control of the light bullet and renders the planar WGA device a viable technology for all-optical processing applications. Indeed, one can envision input and output ports directly coupled to the slab waveguide which could be used to route and process optical data streams and signals.

The outline of the paper is as follows: In Sec. 2, the governing equations and the parameters
in the model are given. The control and routing of single bullets are explored in Sec. 3. In Sec. 4, the combination of nonuniform gain and well as gain-mediated interactions between multiple bullets are studied. Lastly, Sec. 5 contains the concluding remarks and a technological outlook for the WGA device.

2. Governing Equations

The formation of mode-locked light bullets is driven by the competition between spatial diffraction, the Kerr nonlinearity, the saturable absorption created by the nonlinear mode-coupling of the planar waveguides, and the saturable gain applied to the system [6–9]. The slab waveguide array mode-locking model (SWGAML) describes the envelopes of the transverse fields in the 0th, 1st, and 2nd waveguides subject to the physical effects of diffraction, the Kerr nonlinearity, three-photon absorption, saturating gain, attenuation, and waveguide coupling [13].

As previously shown [13], the mode-locking of light bullets is self-starting from white noise when a uniform gain is applied to the system. The inclusion of a non-uniform gain medium creates new classes of mode-locked solutions capable of spatial movement. The governing equations with the nonuniform gain are given by [13]:

\[
\begin{align*}
    i \frac{\partial A_0}{\partial t} + \frac{D}{2} \nabla^2 A_0 + \beta |A_0|^2 A_0 + CA_1 + i \gamma_0 A_0 - i g(x,y,t) \left(1 + \tau \nabla^2 \right) A_0 + i \rho |A_0|^4 A_0 &= 0 \quad (1a) \\
    i \frac{\partial A_1}{\partial t} + C(A_0 + A_2) + i \gamma_1 A_1 &= 0 \quad (1b) \\
    i \frac{\partial A_2}{\partial t} + CA_1 + i \gamma_2 A_2 &= 0 \quad (1c)
\end{align*}
\]

where \( \nabla^2 = \partial_x^2 + \partial_y^2 \) and the saturating gain is given by

\[
g(x,y,t) = \frac{2g_0}{1 + |A_0|^2/e_0} f(x,y,t). \quad (2)
\]

\( A_0, A_1, \) and \( A_2 \) are the envelopes of the normalized electric fields in the 0th, 1st and 2nd waveguide. The parameter \( D \) is the diffraction coefficient which is scaled to be \(-1 \) or \( 1 \) depending upon the sign of the index of refraction. The coupling strength between waveguides is given by the parameter \( C \), and the Kerr nonlinearity strength is described by the parameter \( \beta \). At high-intensities, the parameter \( \rho \) models the the nonlinear loss due to three-photon absorption. The saturating gain is described by \( g \), which depends upon the level of charge carrier injection and the energy of all of the bullets in the system. The parameter \( g(t)\nabla^2 \) term provides a bandwidth limitation on spatial modes as described by the parameter \( \tau \). The presence of charge carrier diffusion in the semiconductor rate equations implies this term should exist [29].

The key contribution of this paper, extending upon previous findings [13], is to consider the spatial-temporal function \( f(x,y,t) \) that contains all of the non-uniformity in the applied gain. The ability to control this parameter is what makes the SWGAML an ideal all-optical processing device. Further, the fine control of the applied gain in both space and time can be readily achieved with today’s modern technological tools [28], thus enabling remarkable potential for all-optical, photonic applications.

The model in (1) represents a simplified version of the full SWGAML. Note that the governing equations for the 1st and 2nd waveguide, (1b) and (1c) respectively, do not contain any diffraction, Kerr, or three-photon absorption terms. As was shown in Kutz and Sandstede [30], the 1st and 2nd waveguides inherit the shape of their modes and their dynamics from the 0th waveguide. This occurs even when a non-uniform gain is applied to the system, thus satisfying the conditions under which the SWGAML governing model (1) holds.
3. **Bullet Routing and Control**

The control and movement of light-bullets is accomplished through manipulation of the gain. The use of gain is necessary because gain is the only parameter which would be convenient to change experimentally. With an experimental setup similar to the schematic shown in Fig. 1, it should be relatively simple to produce a linearly sloped gain, gain ramps with regions of no gain, and even time dependent gain simply by changing how much current is injected into each of the gold layer partitions. The light bullets travel towards regions of higher gain, and this behavior can be exploited in order to control the bullets. Mathematically, this method breaks the translational invariance in the system to manipulate the location and movement of the light bullets.

3.1. **Bullet Stability**

The use of gain as a method for routing the pulses produces a potential complication because light bullets are only stable for a specific range of gain values [13]. If the gain is too small, the bullet will decay. On the other hand, if the gain is too large the bullet will form a breather solution and may even split into multiple bullets. While the ability to destroy a bullet is useful, the breather solutions radiate energy which may cause complications elsewhere in the system. Thus, precision control of the gain level is very important.

For a given range of parameters, solutions to (1) may be computed numerically using collocation methods and assuming

\[ A_n = a_n e^{-i\Theta t} \]  

where \( \Theta \) is the propagation constant and for simplicity, \( A_n \), is assumed to have radial symmetry. In practice, radial solutions are the only light bullets observed in simulations. Using the following parameter values [13]

\[ (D, \beta, C, \gamma_0, \gamma_1, \gamma_2, \rho) = (-1, 8, 10, 0, 0, 10, 1) \]

\[ f(x, y, t) = 1, \]  

radial solution can be computed as in Williams and Kutz [13]. In this previous work, construction of these solutions and their stability where considered via full numerical simulations of the SWGAML.

In our present analysis, we extend the previous findings by explicitly computing the stability of the radially symmetric branch of solutions. This is done by linearizing the governing SWGAML equations (1). In one-dimension, the linear stability is explicitly given Kutz and Sandstede [8]. Extending this method to the radially symmetric equations for the two-dimensional model allows us to numerically compute the linear stability of the light bullet solutions as a function of the input gain parameter \( g_0 \). Indeed, the radially symmetric branch of solutions and the linearized eigenvalues (spectra) can be computed as shown in Fig. 3. Thus
the stable regions of light bullet formation can be computed. Further, if the gain is increased, a Hopf bifurcation can occur, leading to the formation of breathing light bullet solutions [13]. All of this is explicitly characterized via the linear stability analysis.

The plot of pulse height as a function of \( g_0 \) shown in Fig. 3 was obtained using the software package AUTO’s continuation capabilities [31]. The individual spectra were obtained by taking solutions provided by AUTO and computing the linearized spectra using the Chebyshev polynomial representation of the solution [32]. This method provides superior (spectral) accuracy relative to standard finite-difference schemes. Due to phase invariance in the system, the eigenvalue near the origin is known to be exactly zero and therefore it was not used in determining the stability of the operator. However, the magnitude of the numerically computed zero eigenvalue was at worst on the order of \( 10^{-8} \), so the elements of the discrete spectrum appear to be quite accurate.

With this analysis, the different operating regimes of the waveguide array can be determined for a given set of parameters. While the constant gain case will no longer be considered, the majority of the non-uniformity in gain can be considered a perturbation of the uniform gain case. Therefore, knowledge of the uniform gain case gives valuable qualitative and quantitative insight into the operation of the system and possible routes to instability even with nonuniform gain.

### 3.2. Linearly Sloped Gain

The simplest deformation of the uniform gain case is perturbing it with a small linear slope. This choice of non-uniformity breaks the translational invariance found in the uniform case, creating locations of higher gain which are more energetically favorable for the bullet.

When starting from initial white noise, the sloped gain creates regions where bullets are more likely to form. However, when starting from a pre-existing bullet, the saturation of the gain makes the formation of a second bullet impossible. Instead, the bullet itself translates to the region of higher gain. The velocity of the bullet is directly related to the slope of the gain. Using the parameters in (4) with

\[
 f(x,y,t) = 1 + mx
 \]  

(5)

the results in Fig. 4 were obtained. In agreement with physical intuition, the larger the value of \( m \), the larger the speed (translation) of the bullet. Additionally, the velocity of the bullet is

![Fig. 3. On the left, a plot of pulse height as a function of \( g_0 \). Linearly stable regions are shown in blue and linearly unstable regions in red. The three plots on the right show the spectrum of the linearized operators for three labeled points. Points in red are in the right half plane while points in blue are either in the left half plane or are known to be exactly zero.](image)
always in the direction of the gradient of the gain. Using this method, it is possible to control both the position and the velocity of the bullet by appropriately selecting the slope of the gain.

In the numerical results from Fig. 4, the largest $m$ value used is $m = 0.02$. In principle, even larger values of slope could be used. However, large values of $m$ can produce regions where a second bullet forms. The second bullet further saturates the gain and destroys the original bullet. While this may be a physically valid form of movement, it is not the type we are considering here, i.e. the control and manipulation of individual light bullets. If one envisions input and output ports at the edges of the SWGAML, then the gain slopping allows for the ability to move the light bullets to a desired output port for further processing or optical transmission.

### 3.3. Pulse Routing

In addition to simple linear gain ramps, regions of no gain may also be included. Physically, this is achievable by removing the segments of the gold contact layer, creating regions that receive no current injection. Light bullets are a gain dependent phenomena and therefore cannot enter regions with no gain. The no-gain regions then act as barriers and can be used to confine light bullets to a particular region. These regions can be used to create photonic wires. Combining these wires with linearly sloped gain allows for a simple yet robust method of pulse routing.

In order to demonstrate this method of pulse routing, we use this combination to produce a cross shaped junction with an input in one arm and outputs in the other three arms. The gain profile used to create the junction is

\[
 f(x,y,t) = \begin{cases} 
 (1 + mx + ny) & \text{if } |x| < 8 \text{ or } |y| < 8 \\
 0 & \text{otherwise}
\end{cases}
\]  

(6)

The choice of $m$ and $n$ vary and control the bullet by creating preferential directions for bullet motion.

An example of using this gain profile for pulse routing is shown in Fig. 5. Additionally, the first movie (Media 1) shows the evolution of the bullets as a function of time. Here $m = 0.01$.
and $n = 0.01$ (output 1), 0 (output 2), and $-0.01$ (output 3) for routing the pulse up, right, and down respectively. Notice that the photonic wires force the route of the light bullet to pass through the cross shaped junction instead of simply traveling in a straight path.

The combination of no-gain regions with an overall sloped gain allows the construction of photon wires. Additionally, since the level of current injected into any one particular region can be changed electronically, the routing of a particular bullet is accomplished via those same electronics. This is a simple routing mechanism that can be controlled electronically as packets of information traversing the SWGAML can be directed as necessary. Thus this simple photonic wire configuration would allow a single input port to direct data streams to three different output ports for further processing or optical transmission.

3.4. Time-Dependent Gain

In the previous case, stationary gain profiles were used to route the pulse in a variety of complex ways. However, time-independent profiles by their very nature, are not capable of creating all possible bullet trajectories. For instance, it is not possible to produce a periodic orbit using a static profile: a periodic orbit requires a gain profile that makes closed path where the directional derivative along the path is always positive. No continuous function can satisfy this constraint. As stated before, it appears the bullets have effectively no momentum so “leaping” over a discontinuity is also not a valid strategy.

The gain profile may be controlled via external electronics to produce a time-dependent gain. While marginally more complicated technologically, this approach allows a simpler spatial gain profile to be used to generate complex results. To produce a circular orbit, the following gain profile was used:

$$f(x,y,t) = \exp \left(-\alpha \left((x-10\cos(\omega t))^2 + (y-10\sin(\omega t))^2\right)\right)$$

where numerically $\alpha = 0.001$ and $\omega = 2\pi/4000$. This simple time-dependent profile is a Gaussian that is translated in a circle. The value of $\alpha$ controls how strongly the gain profile traps the bullet. The larger $\alpha$ is, the stronger the trapping of the bullet. However, $\alpha$ must also be small enough so that the majority of the bullet receives gain.

Using the profile in (7), the bullet was moved in a circle of radius ten as shown in Fig. 6 and the second movie (Media 2). The red line shows the center of the Gaussian gain profile as a function of time while the white dashed line shows the position of the bullet where the
center-of-mass computation

\[
\text{pulse center} = \frac{\iint x|A_0|^2 \, dx \, dy}{||A_0||^2} \hat{x} + \frac{\iint y|A_0|^2 \, dx \, dy}{||A_0||^2} \hat{y}. \tag{8}
\]

was used to determine the bullet and gain center. As shown in Fig. 6, the difference between the center of the gain and the light bullet peak are nearly indistinguishable to the eye. There is a small lag between the center of the gain and the position of the bullet, but it is negligible compared to the size of the bullet. Since the bullet follows the center of the gain profile, far more complicated trajectories than a simple circle are possible. Indeed, with the sophistication of modern electronics, an addressable array of gain segments can produce almost any desired or routing in the SWGAML.

While time-independent gain profiles are most likely the simplest solution, they are not as flexible as time-dependent gains. The inclusion of time-dependent gains, even with very basic spatial profiles such as a Gaussian, allows more flexibility in the bullet trajectory, including periodic orbits and paths that cross back upon themselves.

4. Multiple Bullet Interaction

Single bullet manipulation is limited primarily to bullet routing. The introduction of additional pulses allows for more complicated dynamics due to bullet interactions. There are two primary methods through which bullets interact. The first is through overlaps of the electric field envelopes. This method requires the bullets to be in close proximity and is capable of changing the location of the pulse center. However, it is sensitive to the relative phase difference and physical separation between the bullets.

While the bullet-to-bullet (via proximity) form of interaction can be used for the manipulation of light-bullets, the sensitivity to phase difference makes it difficult to exploit in practice. The second method for bullet manipulation, gain mediated interactions, is not sensitive to the relative phase differences of the light bullets nor does it depend upon the spatial proximity of the bullets. The equation for gain, (2), saturates when the energy in the system as a whole increases. Therefore, a change in any single bullet impacts the entire system by changing the gain seen by the entire array. Since this interaction only occurs via gain, it is less powerful than the bullet-to-bullet interaction. However, it is more robust. Indeed, the only constraint is that the bullets need to be close enough for this gain model to be valid. Nonetheless, this gain-mediated interaction in combination with the bullet routing discussed in Sec. 3 is sufficient to create logic gates. Additionally, these gates could have input and output ports at the edges of
Fig. 7. All four possible inputs for the WGA acting as a NOR gate, the initial condition is shown on top and the result after pulse interaction is shown on the bottom. The bullet in the region labeled clock acts as a clock signal. If the clock bullet reaches the right hand of the domain, the result is considered "high" otherwise it is considered "low". All four potential logic inputs are tested and the resulting outputs are consistent with a NOR gate.

the WGA, which is desirable from a technological standpoint.

4.1. NOR Gate

The NOR gate is the first logic gate that can be produced using gain-mediated multiple-bullet interactions. Along with the NAND gate, this particular gate is a so-called “master gate” and as such, all other logic gates may be produced through combinations of this gate.

The NOR gate can be generated from three closely spaced strips of gain. As shown in Fig. 7, the first gain strip corresponds to a clock pulse and the other two gain strips are the logic gate inputs. The strips of gain are linearly sloped such that any pulse will translate from left to right. The output is dictated by the clock pulse. If the clock pulse reaches the right hand side of the domain, this is considered the logical high output. Otherwise, the output is logical low.

The key to this gain-mediated interaction process is uneven levels of gain. The system is given only enough gain to support a single bullet. If the initial condition is only the clock pulse, the clock pulse will translate to the right producing a logical high. However, the initial condition may contain up to three bullets. With insufficient gain to support all the bullets, they rapidly decay in size. The rate at which the bullet decay is inversely related to the gain given to a bullet. Therefore, by giving the clock pulse a slightly lower gain than the input pulses, the clock pulse will be annihilated before either of the two input pulses. Therefore, in all other cases the clock pulse is destroyed and one of the two input bullets will translate to the right producing a logical low.

Note that this logic gate has been produced simply by bringing three bullets into close enough proximity that the gain model is valid. No extra electronics or additional control structures are required. This creates a simple yet robust version of a NOR gate.

4.2. NAND Gate

The other master gate is the NAND gate which has also been implemented in the SWGAML. In contrast to the NOR gate, the NAND gate is generated with time-dependent gain profiles.
Fig. 8. All four possible logic inputs for the WGA configured as a NAND gate. The initial condition is a clock and auxiliary pulse along with the different inputs to the system. The output of the system is determined solely by whether or not the clock pulse translates to the right hand side without being destroyed. (Media 3)

Like the previous example in Fig. 6, a Gaussian gain profile is used for each of the clock or input bullets of the system.

Figure 8 shows all four possibilities for the NAND gate. The NAND gate is constructed out of four bullets: a clock bullet, an auxiliary bullet, and two input bullets. Regardless of whether or not the input bullet exists, a translating Gaussian gain profile is generated where the bullet should be. The gain profile will translate the bullet, if one exists, otherwise it will do nothing.

Like the NOR, enough gain for only two bullets is provided to the system. If both inputs are logical low, then the clock and auxiliary bullets are supported by the system and will translate along with the gain profile. If any of the inputs are logical high, then at least one of the pulses will decay. The level of gain provided to each of the bullets is, from lowest to highest, the auxiliary pulse, the clock bullet, input one, and input two. Therefore, if one input is high then the auxiliary bullet will be destroyed while the clock and input bullet survives. This produces a logical high. However, if both inputs are high, then both the clock and auxiliary bullets decay producing a logical low. The dynamics of these processes are illustrated in the third movie (Media 3).

This procedure is similar in concept to the NOR gate, but the time-dependent method provides a measure of extra flexibility. Note that the same approach could be used to create a NOR gate if the gain given to the system is sufficient to support only one bullet. Therefore, with a very simple external change, this system can toggle between two different master gates.

In principle, a NAND gate may be constructed from the same static elements as the NOR gate. However, for practical reasons this is difficult to implement. As stated previously, with sufficiently high gains it is possible for bullets to change positions by forming a second bullet and having the first bullet decay. With gains large enough to support a pair of bullets combined with sloped gain, this happens very frequently in numerical simulation. Though an appropriate value of slope should exist, lower slopes correspond with lower speeds as shown in Fig.
4. Therefore, this problem becomes difficult to analyze numerically as the simulation times become large. This makes the exploration of parameter space through the full governing equations untenable. The time-dependent gain circumvents this issue and allowing both gates to be readily produced. Regardless, through simple concepts and well-known electronics technology, fully operational photonic logic devices can be constructed.

5. Conclusions and Outlook

There is little doubt that light bullets hold tremendous potential for evolving into a critically import technology in the photonics arena. The myriad of research groups around the world working on light bullet stabilization and control attest to their growing importance in the scientific community [10–13, 17–27]. Indeed, there are numerous methods currently proposed for engineering what may eventually evolve into a robust photonics technology. As with all technologies, the successful implementation of light bullet engineering relies on the ability of the system to reliably and inexpensively produce and control the localized optical structures. Using slab waveguide arrays with a fully addressable gain in time and space, we have demonstrated theoretically the ability of the SWGAML to stabilize and manipulate light bullets using simple, electronically addressable gain. The theoretical and computational results show a robust and easily manipulated dynamics. From this, we have demonstrated the operation of both master logic gates: NAND and NOR. From these, all other photonic logic and switches can be constructed, thus suggesting that the SWGAML merits serious consideration as a next generation photonics device. Indeed, this method of generating light bullets has many advantages over other technologies including its robustness, self-starting behavior, and easily addressable routing and interactions via the electronically controlled gain dynamics. Further, the SWGAML architecture relies on simple input and output coupling directly at the edges of the zeroth waveguide. This allows for an arbitrary ability to route and control all-optical data streams.

From a mathematical point of view, explicit stability calculations have been performed on the radially symmetric light bullet structures. The stability analysis shows that there is a robust range of operation for the light bullet evolution. Above a critical gain threshold, the light bullet solutions are stabilized and act as global attractors to the underlying SWGAML system. Further increase in the gain acts to destabilize the light bullets by first inducing a Hopf (breathing) bifurcation before an eventual pulse splitting. Multiple pulses are shown to interact via the gain and the overall cavity energy, creating a distance and phase independent interaction of the pulses. This interaction can form the basis of the logic operations using light bullets.

Acknowledgements

The authors would like to thank Steven Cundiff, Darren Hudson, Mingming Feng, Rich Mirin and Kevin Silverman for discussions related to the waveguide arrays and practical implementations of the gain. J. N. Kutz also acknowledges support from the National Science Foundation (NSF) (DMS-0604700) and the U.S. Air Force Office of Scientific Research (AFOSR) (FA9550-09-0174).